Name: Key Date: _____ Hour: _____

GEO Notes: Isosceles and Equilateral Triangles- Algebra Based

Properties of Isosceles Triangles An **isosceles triangle** has two congruent sides. The angle formed by these sides is called the **vertex angle**. The other two angles are called **base angles**. You can prove a theorem and its converse about isosceles triangles.

- If two sides of a triangle are congruent, then the angles opposite those sides are congruent. **(Isosceles Triangle Theorem)**
- If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

If $\overline{AB} \cong \overline{CB}$, then $\angle A \cong \angle C$.

If $\angle A \cong \angle C$, then $\overline{AB} \cong \overline{CB}$.

Example 1. *Find x and the measure of each side of equilateral triangle RST.*

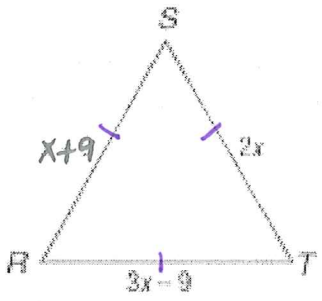
SR = ST def of equilateral Δ

x + 9 = 2x

9 = x

ST = 18
SR = 18
RT = 18

You check your work so these are all ≅



Example 2. *Find x and the measure of each side of isosceles triangle EFG.*

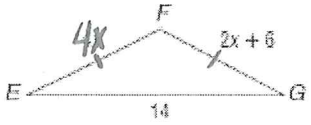
EF = FG def of isosc. Δ

4x = 2x + 6

2x = 6

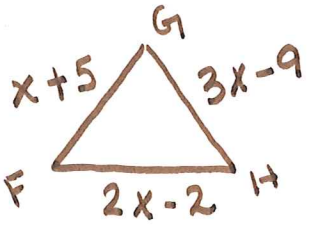
x = 3

EF = 12
FG = 12
EG = 14



Directions: Find x and the measure of each side of the triangle.

3. ΔFGH is equilateral with FG = x + 5, GH = 3x – 9, and FH = 2x – 2.



FG = GH def of eq. Δ

x + 5 = 3x − 9

14 = 2x

7 = x

X= 7

FG= 12

GH= 12

FH= 12

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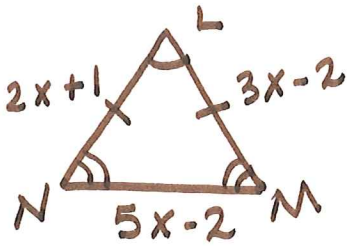
4. ΔLMN is isosceles, <L is the vertex angle, LM = 3x – 2, LN = 2x + 1, and MN = 5x – 2.

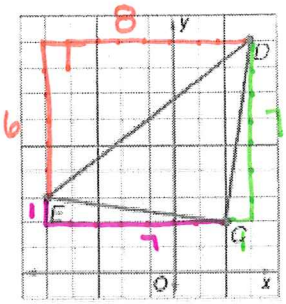


LM = LN
3x - 2 = 2x + 1
x = 3

X= 3 LM= 7
LN= 7 MN= 13

5. **COORDINATE GEOMETRY** Find the measures of the sides of △*DEC*. Classify the triangle by sides.

Use the Distance Formula to find the lengths of each side.



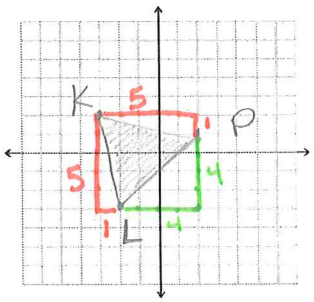
$DE^2 = 6^2 + 8^2$
$DE = \sqrt{100}$
$DE = 10$

$EC^2 = 1^2 + 7^2$
$EC = \sqrt{50}$
$EC = 5\sqrt{2}$

$CD^2 = 1^2 + 7^2$
$CD = \sqrt{50}$
$CD = 5\sqrt{2}$

EC ≅ CD ∴ by def △DEC is an isosceles △.

6. Find the measures of the side of ΔKPL and classify the triangle by its sides.
K(-3,2), P(2,1), L(-2,-3)



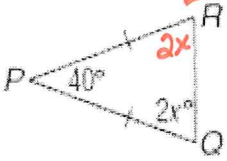
$KL^2 = 5^2 + 1^2$
$KL = \sqrt{26}$

$KP^2 = 5^2 + 1^2$
$KP = \sqrt{26}$

$PL^2 = 4^2 + 4^2$
$PL = 4\sqrt{2}$

KL ≅ KP ∴ by def △KPL is an isosceles △.

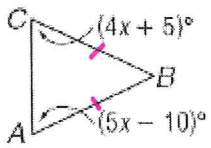
7. Find x.



base <s of isos △ ≅

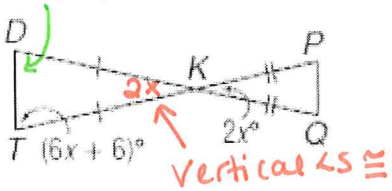
<P + <R + <Q = 180 △ sum
40 + 2x + 2x = 180
4x + 40 = 180
4x = 140
x = 35°

8. Find x. If BC≅BA.



<C ≅ <A
base <s of isosc △ are ≅
4x + 5 = 5x - 10
15 = x

9. Find x.

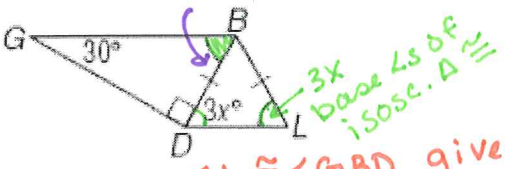


6x + 6 base <s of isosc △ ≅

<D + <DKT + <T = 180 △ sum
6x + 6 + 2x + 6x + 6 = 180
x = 12

Vertical <s ≅

10. Find x.



180 - 30 - 90 = 60° △ sum

3x base <s of isosc. △ ≅

<L ≅ <GBD given
3x = 60
x = 20°