Permutations and Factorials

How many 4 digit pins can be made using the digits 0-9?

Do I provide a sample space?

How do we calculate this?

How many 4 digit pins can be made using the digits 0-9? (Numbers can't be used more than once)?

How many 10 digits numbers are there?

This is a factorial! 10!

Vocabulary:

Multiplication Counting Principle:

Permutation:

Factorial:

Try some with your calculator: Find the value of each.

a. 4!

b. 0!

c. 7!

d. ${}_5P_2$

e. ${}_{10}P_1$

f. ${}_6P_6$

A factorial can also be used when there is repetition.

Example:

How many arrangements are there of the letters in the word "ALGEBRA"

Find the number of outcomes and Tell which method you are using

2) Diana is getting ready for prom. She has her dress, 2 pairs of shoes to pick from, 4 necklaces to pick from and if she is going to wear nylons or not. How many ways can she look?

3) On my new Apple watch, it came with 10 free songs from I-tunes.

   a. How many ways can I play 5 songs?

   b. How many ways can I play all 10 songs?

   c. Out of the 10 free songs, I only keep 6, how many ways can I play all 6?

4) How many arrangements are there of the letters of each of the words.

   a. MULTIPLY

   b. COUGARS

   c. DAKOTA

   d. MACOMB

   e. MATHEMATICS

5) How many ways are there to assign a President and Vice-President from 20 members?

6) How many ways are there to get first, second and third place in a race with 10 participants?

7) How many ways are there to get first, second, third, and fourth place in a race with 8 participants?

8) How many ways are there to get to and from school, if there are 5 different routes you may take?

9) How many different outcomes are there to a three digit lottery? (the lottery uses the digits 0 -9)

10) How many different license plates are possible, if it is 3 letters followed by three digits?

11) How many different license plates are possible, if it is 3 letters followed by three digits, but there is no repetition?

12)How many different license plates are possible, if it is 3 letters followed by three digits, but there is no repetition of just digits?

Permutations and Factorials Key

How many 4 digit pins can be made using the digits 0-9?

Do I provide a sample space? No!

How do we calculate this?

$10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$ ways

How many 4 digit pins can be made using the digits 0-9? (Numbers can't be used more than once)?

$10 \cdot 9 \cdot 8 \cdot 7 = 5{,}040$ ways

Pins

How many 10 digits ~~numbers~~ are there? (Numbers can't be repeated)

3,628,800 ways

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

This is a factorial! $10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Vocabulary:

Multiplication Counting Principle: (#1) → multiply the # of outcomes at each event.

Permutation: (#2)

$_{10}P_4 = 10 \cdot 9 \cdot 8 \cdot 7$

$_7P_2 = 7 \cdot 6$

Factorial: (#3)

$10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Try some with your calculator: Find the value of each.

a. 4! = 24

b. 0! = 1

c. 7! = 5,040

d. $_5P_2$ = 20

e. $_{10}P_1$ = 10

f. $_6P_6$ = 720

6!

A factorial can also be used when there is repetition.

Example:

How many arrangements are there of the letters in the word "ALGEBRA"

7 letters in Algebra

A is repeated twice → $\frac{7!}{2!}$ = 2,520 ways

Find the number of outcomes and Tell which method you are using

2) Diana is getting ready for prom. She has her dress, 2 pairs of shoes to pick from, 4 necklaces to pick from and if she is going to wear nylons or not. How many ways can she look?

MCP $1 \cdot 2 \cdot 4 \cdot 2$ = 16 looks

3) On my new Apple watch, it came with 10 free songs from I-tunes.

a. How many ways can I play 5 songs? (can not repeat songs)

Permutation

$_{10}P_5$ = 30,240 ways

b. How many ways can I play all 10 songs?

factorial

10! = 3,628,800 ways

c. Out of the 10 free songs, I only keep 6, how many ways can I play all 6? 6!

factorial

720 ways

4) How many arrangements are there of the letters of each of the words.

a. MULTIPLY $= \frac{8!}{2!} = 20,160$ ways

b. COUGARS $= 7! = 5,040$ ways

c. DAKOTA $\frac{6!}{2!} = 360$ ways

d. MACOMB $\frac{6!}{2!} = 360$ ways

e. MATHEMATICS $\frac{11!}{2!2!2!} = 4,989,600$ ways

5) How many ways are there to assign a President and Vice-President from 20 members?

$_{20}P_2 = 20 \cdot 19 =$ 380 ways

6) How many ways are there to get first, second and third place in a race with 10 participants?

$_{10}P_3 = 10 \cdot 9 \cdot 8 =$ 720 ways

7) How many ways are there to get first, second, third, and fourth place in a race with 8 participants?

$_{8}P_4 = 8 \cdot 7 \cdot 6 \cdot 5 =$ 1,680 ways

8) How many ways are there to get to and from school, if there are 5 different routes you may take?

MCP $= 5 \cdot 5 =$ 25 ways

9) How many different outcomes are there to a three digit lottery? (the lottery uses the digits 0 -9)

MCP $10 \cdot 10 \cdot 10 =$ 1,000 ways

10) How many different license plates are possible, if it is 3 letters followed by three digits?

MCP $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 =$ 17,576,000 ways

11) How many different license plates are possible, if it is 3 letters followed by three digits, but there is no repetition?

$_{26}P_3 \cdot {}_{10}P_3$ $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 =$ 11,232,000 ways

12) How many different license plates are possible, if it is 3 letters followed by three digits, but there is no repetition of just digits?

MCP $26 \cdot 26 \cdot 26 \cdot 10 \cdot 9 \cdot 8 =$ 12,654,720 ways